Why Moments (and Generalized Moments) Are Used in Statistics and Why Expected Utility Is Used in Decision Making: A Possible Explanation

R. Noah Padilla and Vladik Kreinovich

Department of Computer Science, University of Texas at El Paso
El Paso, TX 79968, USA, rnpadilla2@miners.utep.edu, vladik@utep.edu

1. Formulation of the Problem

- In this talk, we provide a new explanation of two seemingly unrelated phenomena.
- The first is that moments, and, more generally, generalized moments, $M = \sum_{i=1}^{n} p_i \cdot f(x_i)$ are effectively used in statistics.
- E.g., they help decide which approximate model is more accurate.
- The second is that expected utility $\sum_{i=1}^{n} p_i \cdot u(x_i)$ is effectively used in decision making to decide which action is better.
- For moments, x_i are different observed values.
- For decision making, x_i are the outcomes of different situations.
- In both cases, p_i are the probabilities of these values or situations.

2. We Need Faster Computations

- In many computational problems, computation time is still a big problem.
- A natural way to speed up computations is to parallelize computations:
  - first, several processors process data,
  - then they process the results of the first-stage processing, etc.
- To speed up computations, we need:
  - to minimize the number of stages and
  - to minimize the time needed for each stage.
- On each stage, what each processor in a deterministic computer computes is a function of its inputs.

3. Which Functions Should We Compute on Each Stage

- The simplest – and thus fastest to compute – are linear functions.
- However, if only use linear stages:
  - the result will still be a linear function of its inputs, and
  - many real-world dependencies are nonlinear.
- Thus, on some stages, we need to compute nonlinear functions.
- In general, the fewer inputs, the faster it is to compute a function.
- Thus, the fastest to compute are functions of one variable.
- So, on each stage, each processor computes either a linear function, or a function of one variable.

4. Consequent Stages

- It makes no sense to have two consequent stages computing linear functions.
- Indeed, the resulting composition of linear functions is also linear, and it can this be computed in a single stage.
- It also does not make sense to have two consequent stages computing functions of one variable.
- Indeed, the composition of such functions is still a function of one variable.
- Thus, to speed up computations, we need consequent stages to be different.

5. How Many Stages Do We Need

- The fastest is when we use only one stage.
- However, in this case, we:
  - either compute a linear function – while many real-life functions are nonlinear
  - or use only one input – while we want to take all the inputs x_i into account.
- So, we need at least two stages.

6. Two Stages: First Option

- Due to the above, these stage must be different.
- If the first stage is linear and the following one nonlinear, then, in general, we compute a function

$$f\left(a_0 + \sum_{i=1}^{n} a_i \cdot x_i\right).$$

- Comparing such values is equivalent to comparing the corresponding linear combinations $a_0 + \sum\limits_{i=1}^{n} a_i \cdot x_i$.

7. Two Stages: Second Option

- If the first stage is nonlinear and the second one linear, then we compute expressions $a_0 + \sum_{i=1}^{n} a_i \cdot f_i(x_i)$.
- This provides a more general opportunities for comparison.
- In particular:
  - if a priori, we have no reason to prefer some i's,
  - then it makes sense to use the same nonlinear function $f_i(x) = f(x)$ to process all the inputs.
- Thus, we get the expression $a_0 + \sum_{i=1}^{n} a_i \cdot f(x_i)$.
- This is exactly what is used when we use generalized moments or expected utility.
- Thus, we have indeed explained the desired expressions.

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